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Interference phenomena in the photon production between two oscillating walls

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Abstract

We study the photon production in a 1D cavity whose left and right walls oscillate with the frequency Ω_L and Ω_R, respectively. For $\Omega_L \neq \Omega_R$, the number of generated photons by the parametric resonance is the sum of the photon numbers produced when the left and the right wall oscillates separately. But for $\Omega_L = \Omega_R$, the interference term proportional to $\cos\phi$ is found additionally, where ϕ is the phase difference between two oscillations of the walls.

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The standard Casimir effect [1] indicates the presence of vacuum fluctuations of the electromagnetic field. The modified boundary conditions of the field, which characterize the dynamical situation, change zero-point vacuum fluctuations inside a perfectly reflecting cavity and give photon productions [2]. This phenomenon has been extensively studied when the one of the walls oscillates [3–12]. Photon productions by parametric resonance when the wall of the cavity oscillates with one of the resonant field frequencies, give one a possibility to observe significant effects under experimental situations and to obtain quantitative results [9–11].

Recently, Lambrecht *et al.* [13] studied the radiation emitted by two oscillating walls using the scattering approach. Therein, boundaries oscillate symmetrically with respect to the center of the cavity or globally with the length of cavity kept constant. Resonance enhancement occurs when the oscillating walls have even integer multiples of the fundamental frequency of the cavity for the former and odd integer multiples for the latter. If we adopt the perturbation approach of Ref. [12] the above results can be easily obtained. Moreover, we can further proceed to a more general situation where the walls oscillate with different frequencies and with a phase difference.

In this Rapid Communication, we shall present the interference phenomena in the number of parametrically generated photons when two walls oscillate with the same frequency but with a phase difference. Further, we shall show that the result of Ref. [13] can be explained by a constructive interference or a destructive interference. For this purpose, we shall calculate the number of produced photons by the parametric resonance in a cavity with two oscillating walls. For the different frequencies, there is no interference and the number of generated photons by the parametric resonance is the sum of the photon numbers produced when the left and the right wall oscillates separately.

We start with a field operator $A(x,t)$ associated with a vector potential which satisfies the one-dimensional wave equation ($c = 1$)

$$\frac{\partial^2 A}{\partial t^2} - \frac{\partial^2 A}{\partial x^2} = 0. \tag{1}$$

Following Refs. [3,14], we expand the field operator as

$$A(x,t) = \sum_n [b_n \psi_n(x,t) + b_n^\dagger \psi_n^*(x,t)] \tag{2}$$

where ${b_n}^\dagger$ and b_n are the usual creation and annihilation operators. Here $\psi_n(x,t)$ is the mode function which vanishes at the left and the right walls which are located at $x = L(t)$ and $x = R(t)$ respectively, i.e. $\psi_n(L(t),t) = 0 = \psi_n(R(t),t)$. Let us introduce the instantaneous mode basis function

$$\varphi_k(x,t) = \sqrt{\frac{2}{R-L}} \sin\frac{k\pi(x-L)}{R-L}, \tag{3}$$

to evolve

$$\psi_n(x,t) = \sum_{k=1}^{\infty} Q_{nk}(t)\varphi_k(x,t) \tag{4}$$

where k is a positive integer. Now we consider the small motions of the walls ($\epsilon \sim \epsilon_L \sim \epsilon_R \ll 1$)

$$L(t) = \Lambda \epsilon_L(t) \tag{5}$$

and

$$R(t) = \Lambda[1 + \epsilon_R(t)], \tag{6}$$

where Λ is the distance of two walls in the static situation and ϵ is small parameter which characterizes the small deviation of the walls from the initially static position. Considering only to the first order in ϵ, from (1) and the orthogonality of the mode functions we have an infinite set of coupled differential equations

$$\begin{aligned}\ddot{Q}_{nk} = &-{\omega_k}^2 Q_{nk} + 2\omega_k^2[\epsilon_R(t) - \epsilon_L(t)]Q_{nk} \\ &+2\sum_j G_{kj}\dot{Q}_{nj} + \sum_j \dot{G}_{kj} Q_{nj} + O(\epsilon^2)\end{aligned} \tag{7}$$

where

$$G_{kj} = \frac{g_{kj}^R \dot{R} - g_{kj}^L \dot{L}}{\Lambda} \tag{8}$$

and ω_k is the mode frequency for the static walls:

$$\omega_k = \frac{k\pi}{\Lambda}. \tag{9}$$

Here we used the following relations

$$g_{jk}^L \equiv -(R - L)\int_L^R \varphi_k \frac{\partial \varphi_j}{\partial L} dx = \frac{2jk}{k^2 - j^2} \tag{10}$$

and

$$g_{jk}^R \equiv (R - L)\int_L^R \varphi_k \frac{\partial \varphi_j}{\partial R} dx = (-1)^{j+k}\frac{2jk}{k^2 - j^2}. \tag{11}$$

Now we consider the special motion of the walls where the left and the right walls oscillate according to

$$\epsilon_L(t) = \epsilon a_L \sin(\Omega_L t + \phi_L) \tag{12}$$

and

$$\epsilon_R(t) = \epsilon a_R \sin(\Omega_R t + \phi_R), \tag{13}$$

where, without loss of generality, we set $\phi_L = \phi$ and $\phi_R = 0$.

Introducing the new dynamical variable [12]

$$X_{n,k\mp} = \sqrt{\frac{\omega_k}{2}}\left(Q_{nk} \pm i\frac{\dot{Q}_{nk}}{\omega_k}\right), \tag{14}$$

and using the vector notation

$$\vec{X}_n(t) = (X_{n,1-}, X_{n,1+}, X_{n,2-}, \cdots)^T, \tag{15}$$

Eq. (7) is transformed to the following first order differential equation

$$\frac{d}{dt}\vec{X}_n(t) = V^{(0)}\vec{X}_n(t) + \epsilon V^{(1)}\vec{X}_n(t) \tag{16}$$

where $V^{(0)}$ and $V^{(1)}$ are matrices given by

$$V^{(0)}_{k\sigma,j\sigma'} = i\omega_k\sigma\delta_{kj}\delta_{\sigma\sigma'} \tag{17}$$

and

$$V^{(1)}_{k\sigma,j\sigma'} = \omega_1\sum_{s=\pm}\left(v^{Rs}_{k\sigma,j\sigma'}e^{si\Omega_R t} - v^{Ls}_{k\sigma,j\sigma'}e^{si\Omega_L t}\right), \tag{18}$$

where

$$\begin{aligned} v^{As}_{k\sigma,j\sigma'} &= \sigma a_A e^{is\phi_A} \\ &\quad\times\Big[\gamma_A g^A_{kj}\sqrt{\frac{j}{k}}\Big(\frac{\sigma'}{2} + s\frac{\gamma_A}{4j}\Big) - s\frac{k}{2}\delta_{kj}\Big], \end{aligned} \tag{19}$$

with $A = L, R$, and $s, \sigma, \sigma' = +, -$. Here we used $\Omega_A = \gamma_A\omega_1$ and $\omega_k = k\omega_1$.

Taking the following power-series expansion in ϵ

$$\vec{X}_n = \vec{X}^{(0)}_n + \epsilon\vec{X}^{(1)}_n + \epsilon^2\vec{X}^{(2)}_n + \cdots, \tag{20}$$

we have the zeroth order and first order equation

$$\frac{d}{dt}\vec{X}^{(0)}_n = V^{(0)}\vec{X}^{(0)}_n, \tag{21}$$

$$\frac{d}{dt}\vec{X}^{(1)}_n = V^{(1)}\vec{X}^{(0)}_n + V^{(0)}\vec{X}^{(1)}_n, \tag{22}$$

which can be easily integrated to give the following solutions:

$$X^{(0)}_{n,k\sigma}(t) = \delta_{nk}\delta_{\sigma -}e^{-i\omega_k t}, \tag{23}$$

$$\begin{aligned} X^{(1)}_{n,k\sigma}(t) &= e^{\sigma i\omega_k t}\int_0^t dt' e^{-\sigma i\omega_k t'}\sum_{j,\sigma'} V^{(1)}_{k\sigma,j\sigma'}X^{(0)}_{n,j\sigma'} \\ &= \omega_1 e^{\sigma ik\omega_1 t}\int_0^t dt'[v^{R-}_{k\sigma,n-}e^{-i(\sigma k+\gamma_R+n)\omega_1 t'} \\ &\quad + v^{R+}_{k\sigma,n-}e^{+i(-\sigma k+\gamma_R-n)\omega_1 t'} - (R\leftrightarrow L)]. \end{aligned} \tag{24}$$

Using the continuity conditions at $t = T$ we can find easily the Bogoliubov coefficient β_{nk} in the solution

$$\psi_n(x, t > T) = \sum_k\left[\alpha_{nk}\frac{e^{-i\omega_k t}}{\sqrt{2\omega_k}} + \beta_{nk}\frac{e^{i\omega_k t}}{\sqrt{2\omega_k}}\right]\varphi_k(x). \tag{25}$$

For the ϵ-order approximation, it is just the coefficient of the negative frequency $e^{+i\omega_k t}/\sqrt{2\omega_k}$ in the solution

$$Q_{nk}(t) = \frac{1}{\sqrt{2\omega_k}}[X_{n,k-} + X_{n,k+}]. \tag{26}$$

By noting that $\omega_1 T \gg 1$ we take only the dominant terms which are proportional to the time

$$\beta_{nk} = \epsilon\omega_1 T(v^{R+}_{k+,n-}\delta_{k,\gamma_R-n} - v^{L+}_{k+,n-}\delta_{k,\gamma_L-n}). \tag{27}$$

Using (10), (11) and (19), we write explicitly

$$\beta_{nk} = \frac{1}{2}\epsilon\omega_1 T\sqrt{kn}\left[(-1)^{\gamma_R}a_R\delta_{n,\gamma_R-k} - e^{i\phi}a_L\delta_{n,\gamma_L-k}\right]. \tag{28}$$

and using $N_{nk} = |\beta_{nk}|^2$ we can obtain the number of created photons with the frequency ω_k which comes from the initial mode function with frequency ω_n:

$$N_{nk} = N^L_{nk} + N^R_{nk} - 2(-1)^{\gamma_R}\sqrt{N^L_{nk}}\sqrt{N^R_{nk}}\cos\phi, \tag{29}$$

where

$$N^A_{nk} = \left(\frac{1}{2}\epsilon\omega_1 T\right)^2 kna_A^2\delta_{n,\gamma_A-k} \tag{30}$$

with $A = L, R$. By summing over n we finally have the total number of generated photons with the frequency ω_k

$$N_k = N^L_k + N^R_k - (-1)^{\gamma_R}2\sqrt{N^L_k}\sqrt{N^R_k}\cos\phi\delta_{\gamma_L\gamma_R}, \tag{31}$$

where

$$N^A_k = \left(\frac{1}{2}\epsilon\omega_1 T\right)^2 k(\gamma_A - k)a_A^2, \tag{32}$$

or explicitly we have

$$\begin{aligned} N_k = \left(\frac{1}{2}\epsilon\omega_1 T\right)^2 & [k(\gamma_R - k)a_R^2 + k(\gamma_L - k)a_L^2 \\ & -(-1)^{\gamma_R}2k(\gamma_L - k)a_R a_L\cos\phi\delta_{\gamma_L,\gamma_R}]. \end{aligned} \tag{33}$$

Note that this result agrees with our previous result [12] when the only right-side wall oscillates ($a_L = 0,\ a_R = 1$).

Now we consider some special case where the two frequencies of walls are same $\Omega_L = \Omega_R = \Omega$ ($\gamma_L = \gamma_R = \gamma$). If we rewrite (31) directly from (28), we have the following form

$$N_{nk} = \left(\frac{1}{2}\epsilon\omega_1 T\right)^2 kn\left|(-1)^{k+n}a_R - e^{i\phi}a_L\right|^2\delta_{n,\gamma-k}. \tag{34}$$

One easily find that for $\phi = 0$ or π this result corresponds to the situation studied in Ref. [13] [see Eq. (12) in the perfect-mirror limit ($\rho \to 0$)] except for the time-dependence. Therein

they used scattering approach and assumed the linear dependence of time. For the kth mode photon numbers, it follows from (31) that

$$N_k = N_k^L + N_k^R - (-1)^\gamma 2\sqrt{N_k^L}\sqrt{N_k^R}\cos\phi. \tag{35}$$

It is worth noting that this formula resembles the intensity formula of the double-slit interference experiment except for the factor $(-1)^{\gamma+1}$. The last term in (35) is the interference term and the photon numbers vary as the phase difference ϕ changes.

In order to examine the interference pattern we further restrict the situation so that the walls oscillate with the same amplitude ($a_L = a_R = 1$). In this case

$$N_k = \left(\frac{1}{2}\epsilon\omega_1 T\right)^2 2k(\gamma - k)[1 - (-1)^\gamma \cos\phi]. \tag{36}$$

When the walls oscillate with even mode frequency $\Omega = 2\omega_1, 4\omega_1, ...$, the interference effect is characterized by the function $1 - \cos\phi$. Then the number of photons is maximal (constructive interference) at $\phi = \pi$ (the two walls oscillate symmetrically with respect to the center of the cavity) and minimal (destructive interference) at $\phi = 0$ (the walls oscillate together while keeping their distance constant). For $\Omega = 3\omega_1, 5\omega_1, ...$ the number of created photons is proportional to $1 + \cos\phi$ and maximal at $\phi = 0$ and minimal at $\phi = \pi$. As pointed out in Refs. [12,13] the photon distribution shows a parabolic spectrum and hence the maximum value of photon number appears at the nearest mode frequency $\omega_k = \Omega/2$.

For the general cases with different frequencies of the wall ($\Omega_L \neq \Omega_R$), the interference term in (31) vanishes and we have the result of no interference

$$N_k = N_k^L + N_k^R, \tag{37}$$

that is, the number of generated photons by the parametric resonance is the sum of the photon numbers produced when the left and the right wall oscillates separately. Then there are two peaks at the nearest modes of frequencies $\omega_k = \Omega_L/2$ and $\omega_k = \Omega_R/2$.

In summary, we have calculated the number of produced photons by the parametric resonance in a cavity whose left and right walls oscillate with respective frequencies, phases, and amplitudes. For the oscillations of the same frequency, we have presented the interference phenomena where the photon number is a function of the phase difference. But for the different frequencies, there is no interference and the photon number is the sum of the photon numbers produced when the left and the right wall oscillates separately.

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